Influence of Surface Metallization on the Propagation Characteristics of Surface Magnetostatic Waves in an Axially Magnetized Rectangular YIG Rod

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Abstract—It has been shown that the boundary conditions strongly influence the propagation characteristics of surface magnetostatic waves propagating in the direction of the external dc magnetic field; the frequency spectrum narrows and the cutoff wavenumber decreases continuously as the gap between the sample surfaces and metal walls increases in the direction in which the magnetic potential decays exponentially. Experimental results obtained at X band agree well with the theoretical estimates.

INTRODUCTION

MUCH experimental consideration has been given to surface magnetostatic waves in a YIG slab propagating in the direction perpendicular to an external dc magnetic field. Nonreciprocity of the propagation has been demonstrated [1]–[3]. Some publications described surface magnetostatic waves propagating in the direction of the external dc magnetic field in the case of a rectangular YIG rod [4], [5]. Dispersion relations for this case have not been clarified. Joseph and Schlömann derived the dispersion relations of surface magnetostatic waves in an axially magnetized circular YIG rod in a free space propagating in the direction of the external dc magnetic field [6]. Olson *et al.* derived that the upper limit of the frequency spectrum of surface magnetostatic waves changes from $\omega = \omega_0 + \omega_M$ to $\omega = \omega_0 + \omega_M/2$, where ω is the driving angular frequency, ω_0 is the precession angular frequency, and ω_M is the saturation magnetization angular frequency, and that the cutoff wavenumber changes from infinity to a certain value if the surface metallization is replaced with air. They also showed experimental observations of surface magnetostatic waves including a wider passband and increased wavenumbers for metallized YIG rods [7]. Asano and Masuda *et al.* independently extended their theory and derived continuous variations of the frequency spectrum and the cutoff wavenumber as a function of the gap between the surface of a circular rod and the surrounding metal wall [8], [9].

We have derived the dispersion relations of magnetostatic waves in a rectangular YIG rod propagating in the direction of the external dc magnetic field when there is a finite gap between the sample surfaces and metal walls. For surface magnetostatic waves, the frequency spectrum narrows and the cutoff wavenumber decreases as the gap increases in the direction in which the magnetic potential decays exponentially. This behavior is similar to that found in the case of a circular rod. In this paper, it is emphasized that the passband of surface magnetostatic waves is tunable under the constant dc magnetic field with the variation of the spacing of metal walls. We present experimental results of dispersion relations of surface magnetostatic waves at X band with theoretical estimates.

Manuscript received May 14, 1973; revised October 11, 1973.
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THEORY

A rectangular YIG rod of cross section $2a \times 2b$ is placed at the center of a metallic guide of $2d \times 2b$ as shown in Fig. 1. Let the magnetic potential inside the rod be

$$\Phi = (A \cos \alpha x + B \sin \alpha x) \cdot (C \cos \beta y + D \sin \beta y) \exp (\mp ikz). \quad (1)$$

The boundary conditions are such that normal components of magnetic flux density B_x and tangential components of magnetic field H_y must be continuous on the sample surfaces at $x = \pm a$ and that normal components of magnetic flux density B_x and B_y must vanish on the metal surfaces at $x = \pm d$ and $y = \pm b$. From these boundary conditions, a transcendental equation on α is obtained:

$$\tan 2\alpha a\{(\kappa^2 + 1)\beta^2 - \mu(\alpha^2 + \beta^2) + \mu^2\alpha^2 + [(\kappa^2 - 1)\beta^2 - k^2 + \mu^2\alpha^2] \cosh 2(\beta^2 + k^2)^{1/2} \cdot (d - a)\} - 2\mu\alpha(\beta^2 + k^2)^{1/2} \sinh 2(\beta^2 + k^2)^{1/2} \cdot (d - a) = 0 \quad (2)$$

and from Maxwell's equation $\nabla \cdot \boldsymbol{B} = 0$, the characteristic equation is expressed as follows:

$$\mu(\alpha^2 + \beta^2) + k^2 = 0. \quad (3)$$

In (2) and (3), μ and κ are diagonal and off-diagonal elements of the permeability tensor, respectively, and $\beta = n\pi/2b$ (n: positive integers). For surface magnetostatic waves, α^2 must be negative and so $\alpha^2 = -\gamma^2$, $\gamma^2 > \beta^2$. The cutoff frequency of surface magnetostatic waves is given for $\gamma \rightarrow \beta$ in (2) and is expressed as follows if $a \geq b$:

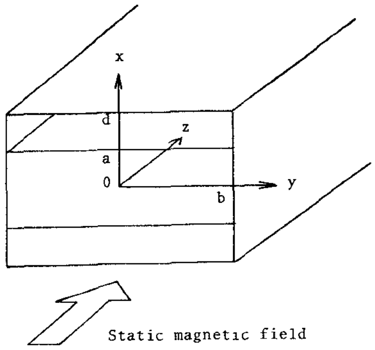

Fig. 1. Metallic waveguide inserted with a rectangular YIG rod.

$$(\kappa^2 - \mu^2 + 1) + (\kappa^2 - \mu^2 - 1) \cdot \cosh \frac{d-a}{b} n\pi - 2\mu \sinh \frac{d-a}{b} n\pi = 0. \quad (4)$$

If the surfaces of the sample are free, $d \gg a,b$, then (4) becomes

$$\omega = \omega_0 + \omega_M/2 \quad (5)$$

and if the surfaces are metallized, $d = a$, then (4) becomes

$$\omega = \omega_0 + \omega_M. \quad (6)$$

The results of (5) and (6) coincide with those in the case of a circular rod. The cutoff wavenumber is given for $\mu \to +0$ in (2) and is expressed as follows in the case of free surfaces:

$$k_c = \left(\frac{\omega_M}{\omega_0}\right)^{1/2} \frac{n\pi}{2b} \quad (7)$$

and in the case of metallized surfaces

$$k_c = \frac{\kappa}{\mu^{1/2}} \frac{n\pi}{2b} \to \infty. \quad (8)$$

The same results as (5)–(8) were already shown by Olson *et al.* [7]. From the above discussion, it is expected that the frequency spectrum and the cutoff wavenumber change as a function of the gap between the sample surfaces and the conducting metal walls. For a small gap, effects of metal plates are strong and the waves with large wavenumbers are allowed in the transmission. For a large gap, effects of metal plates are weak and only the waves with small wavenumbers are allowed.

Due to the demagnetizing field inside the rod, the internal dc magnetic field is not uniform along the direction of propagation. But if the central part of the rod is covered with metal plates of length $2l$ as shown in Fig. 2, the internal dc magnetic field is nearly uniform between $z = \pm l$ along the direction of the external dc magnetic field and an analysis is simplified. The difference of magnetic field strength between the center at $z = 0$ and the cover ends at $z = \pm l$ is 30 Oe for our sample [10].

EXPERIMENTAL PROCEDURES

Our sample was a single crystal YIG rectangular rod of dimension $1.89 \times 2.21 \times 10.67$ mm^3 and all the surfaces were polished optically flat. Microwave power from a

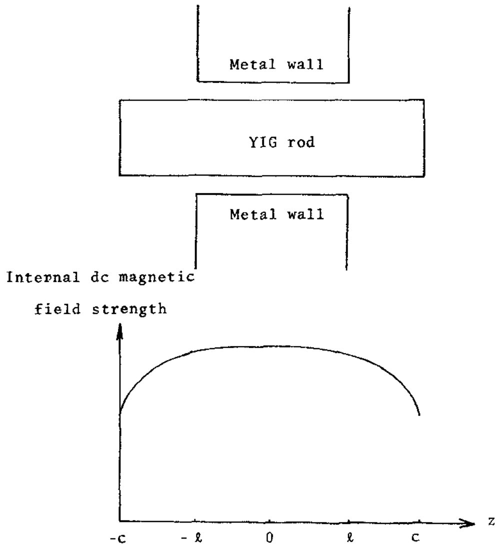

Fig. 2. Coordinate system and internal dc magnetic field strength along the direction of an external dc magnetic field. $2c = 10.67$ mm and $2l = 5.0$ mm.

klystron 2K25B was applied to a thin wire antenna situated near the rod end. The thin wire antenna had an open-circuit end and the rod was located apart $\lambda/4$ from the open end. In another experiment, the YIG rod was completely covered with metal plates between $z = \pm c$. In this case, transmitted microwave power of surface magnetostatic waves saturates with an application of microwave power above 1 mW. On the other hand, that of volume magnetostatic waves increases linearly till the applied microwave power exceeds 10 mW. To prevent saturation effects and instabilities of surface magnetostatic waves, the microwave power was limited to below 0.5 mW. And also, to avoid an unsaturation of magnetization, experiments were performed at X band 9.560 GHz.

Transmitted microwave power or reflected power through a circulator was rectified by the diode 1N23B and applied to a Y input of an X–Y recorder. The X input was the sweep voltage for dc magnetic field. We have determined the modes of traveling surface magnetostatic waves from the spectrum of the transmitted or reflected microwave power versus internal dc magnetic field strength. Our sample holder was rotatable and we have adjusted the propagation direction to that of the external dc magnetic field, by observing the time delay of volume magnetostatic waves. The gap between the sample surfaces and the metal walls was changed by a suitable combination of the Teflon films of 0.05-, 0.1-, 0.15-, 0.2-, and 0.25-mm thicknesses with small tan δ.

EXPERIMENTAL RESULTS AND DISCUSSION

In Fig. 3 are shown the resonance absorption characteristics of volume magnetostatic waves with metallized surfaces. The vertical axis denotes the number of resonating standing waves in the direction of propagation and the horizontal axis denotes the difference of the internal dc magnetic field strength from the turning point $H_{tp} = \omega/\gamma$. Since the wavenumber k is the function of the internal

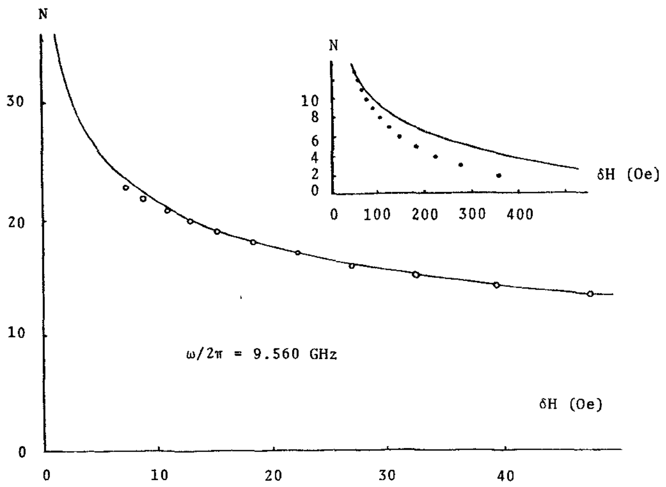

Fig. 3. Peak number N versus field difference δH from the turning point. The turning point is $\omega/\gamma = 3414$ Oe ($\delta H = 0$). Theoretical value of N for (0,1) volume wave (solid line); experimental value (open circle).

dc magnetic field strength H, and H is also the function of the coordinate z, N is given by integrating k from $z = -c$ to $z = +c$ and dividing by π. The theoretical value of N for $(\alpha,\beta) = (0,\pi/2b)$ is given in Fig. 3. The higher modes of N above 20 have small amplitudes in the spectrum and merge in just below the turning point. So, they cannot be distinguished separately. The departures from the theoretical line for the lower modes of N below 10 increases with decreasing mode number N because the sample ends are not metallized in addition to the invalidity of magnetostatic approximations for the long wavelength. It is concluded that the thin wire antenna parallel to the x axis excites the lowest dominant (0,1) mode, because it produces a uniform RF magnetic field. The standing waves versus resonance absorption characteristics of volume magnetostatic waves show small changes as the gap increases, but the resonance absorption occurs within the predicted region.

On the other hand, the absorption characteristics of surface magnetostatic waves show remarkable changes as mentioned below. It is expected that the lowest $n = 1$ mode will be excited. In Fig. 4 are shown the dispersion relations of surface magnetostatic waves as a function of the internal dc magnetic field strength with the gap between the sample surfaces and metal walls as parameter. The length of the shielded portion of the rod is $2l$ and if the following condition:

$$2kl = m\pi, \qquad m: 0 \text{ or positive integers} \tag{9}$$

is satisfied, it is expected that the standing waves resonance absorptions occur. Experimental resonance peaks in the transmitted or reflected spectrum are plotted. Their field strengths were measured from the turning point to avoid the effect due to the anisotropy field of the sample. The experimentally observed fields for resonance are placed on the theoretical H_{res} versus k curves, thus yielding a value for $2kl/\pi = m$. In Fig. 4 the following two features are obvious. First, it may be seen that m converges to integer values as the number of the standing waves in

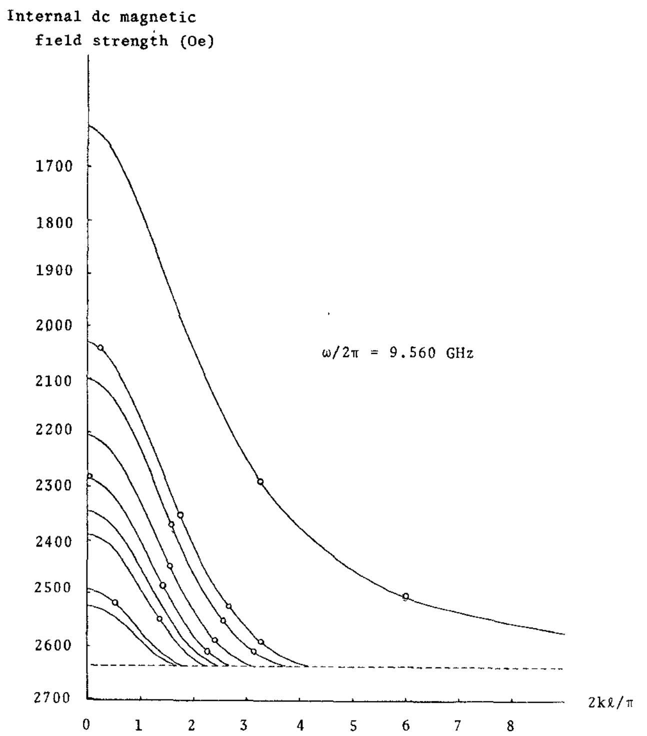

Fig. 4. Dispersion relations of surface magnetostatic waves. The gap d–a is, from the top to the bottom, 0, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 1.0 mm, and ∞, respectively. The broken line is a cutoff dc magnetic field strength. Theoretical value of $n = 1$ surface waves (solid line); experimental value (open circle).

the shielded portion increases. Second, m departs much from integer values as the gap decreases. This fact is qualitatively explained as follows. In the unshielded portion near the rod ends, surface magnetostatic waves become cutoff and do not propagate if the internal dc magnetic field strength in this portion is below 2524 Oe. Even if the internal dc magnetic field strength in the shielded portion is above 2524 Oe, the internal dc magnetic field strength in the unshielded portion can be below 2524 Oe due to the demagnetizing field. Hence, the reflection of surface magnetostatic waves at $z = \pm l$ must be taken into account. It is natural to assume that the upper and lower limit values of the impedance of surface magnetostatic waves are given when the sample surfaces are metallized or free, and the impedance varies continuously as a function of the gap. So the difference of the impedance of propagating surface magnetostatic waves between the unshielded portion and the shielded portion of the rod becomes large and m departs much from integer values as the gap decreases. The same phenomenon has been invoked for a transmission-type cavity resonator. Without coupling holes, the resonant wavelength is a multiple of a half guide wavelength. With coupling holes, the resonant wavelength departs from a multiple of a half guide wavelength.

The driving antenna was located near the rod end and so the excitation of surface magnetostatic waves was weak by a factor of 10 or 100 compared with that of volume magnetostatic waves. Thus the resonances with m of zero are observable for the gap d–a = 0.05, 0.3, and 1.0 mm only. It is impossible to determine the cutoff dc field strength exactly without strengthening the excitation or without amplifying the signal. Using a high-gain amplifier after the detection made the measurement confusing because the leakage microwave power interfering with the signal was also amplified. But it is clear that the dc field range of surface magnetostatic waves becomes narrower as the gap increases.

CONCLUSION

We have theoretically shown that the upper limit of the frequency spectrum of surface magnetostatic waves changes continuously from $\omega = \omega_0 + \omega_M$ to $\omega = \omega_0 + \omega_M/2$ and the cutoff wavenumber changes continuously from infinity to a certain value as the gap between the surfaces and metal walls changes from zero to infinity in the direction in which the magnetic potential decays exponentially. Experimental results obtained at X band agree qualitatively with the theoretical estimates. By using the above results, new types of tunable microwave filters using surface magnetostatic waves are possible. These filters are mechanically tunable in contrast with the usual YIG filters that are tuned by controlling the dc magnetic field.

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Waveguide Sandwich Filters

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***Abstract*—A new class of waveguide filters is introduced, constructed from several thin plates sandwiched together. The combination of alternate plates having large and small rectangular apertures leads to a propagating structure which possesses a bandstop response and prescribed characteristic impedance. This basic element may be used as a simple compact bandstop filter, particularly where the main passband and stopband are well separated, such as in harmonic rejection. For filters with many stopbands, combinations of several waveguide sandwich filter elements are used to provide the main passband and the required attenuation characteristics in the prescribed stopbands. Although the filter is ideally suited for bandstop filtering due to its small size, low cost, low loss, and high power handling capability, additional applications to bandpass filtering and dispersive delay line operation are also cited.**

INTRODUCTION

THE SIMPLEST form of the waveguide sandwich filter [1] is for use in a rectangular waveguide system supporting the dominant H_{01} mode. The laminar construction uses two different plates as shown in Fig. 1. The width a of the rectangular apertures is the same as the width of the waveguide into which the filter is to operate and the two types A and B have different heights b_1 and b_2. Normally, the plates will be of the same thickness to reduce the overall cost of construction.

The overall waveguide filter element is formed from a large number of plates sandwiched together with the plates type A and B alternating. Normally, a rigid assembly is formed by dowelling the plates together and then soldering from the outside. The basic section then formed is shown in Fig. 2, and is characterized by a waveguide of uniform width with periodic transverse slots in the broad walls. Thus, by using an equivalent circuit comprised of a cascade of n identical sections, each of which is formed from a series stub symmetrically embedded in a short piece of uniform waveguide, the overall electrical properties of this basic waveguide sandwich filter element may be deduced. The device is shown to be essentially a bandstop structure with a broad passband at a lower frequency, but above the normal waveguide cutoff frequency.

Manuscript received May 21, 1973; revised October 25, 1973.
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